STUDYING THE PHYSICOMECHANICAL PROPERTIES OF LOOSE MATERIALS

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A physical model is proposed for loose material as a discrete statistical system, and the phase parameters of the properties of the system are analyzed. The rupture resistance of loose material is considered as a function of its packing density.

Loose materials occupy a prominent place among other materials commonly processed in various branches of the popular economy.

It is therefore an important matter to study the physicomechanical properties of loose materials which are required in engineers' calculations. Up to the present time however, most of the results published on this subject have lacked any indication of the range of applicability of the experimental data for various conditions of existence of the loose material. In recent years questions have frequently arisen as to the manner in which the physicomechanical properties of loose materials depend on their leading parameters, such as the packing density, the granulometric composition, the granulomorphological characteristics, and so on.

The representation of loose material as a continuous medium has, under certain circumstances, provided data approximately describing the static properties on the basis of the analytical methods developed for continuous media.

However, theories based on this concept are unable to explain dynamic effects. Dynamic problems occupy a major position in the majority of processes, and, as before, these have to be taken into account by introducing a series of experimental coefficients.

This occurs because existing theories take no account whatsoever of the fundamental property of a loose material – its discrete nature.

Only a physical model of a loose material based on its discrete structure can give a proper representation of real processes. Under certain conditions, the discrete structure of loose material allows a transition to continuity to be made. This transition is an exception and not a rule; it is not general, but peculiar to the loose material, and has to be derived from the general solution.

Loose material is a special variety of a discrete statistical system, subject to the following conditions: there exists a certain minimum elementary volume below which the physical sense of the existence of the loose material loses its meaning; there exists a maximum volume (determined by the height and width of the vessel) above which loose materials acquire the properties of a quasicontinuous medium.

At a specified height of a loose layer, only a finite number of particles may exist. Between neighboring layers of loose particles there is a minimum difference of potential energies. The discreteness of the loose material determines the discrete character of the potential energy spectrum with respect to the height of the layer.

For a layer of loose material we have the relation

$$\Delta l \Delta P \leqslant H. \tag{1}$$

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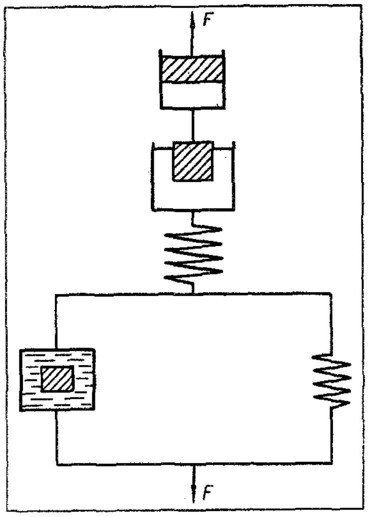

Fig. 1. Fundamental principles of the physical model of a loose material.

TABLE 1. Characteristics of a Statistical Assembly

Serial No.	Parameters	Characteristics of the statistical assembly determined by the parameter
1	Packing density	Mobility of the particles
2	Ratio of the height H of the loose material to the width D of the vessel	Degeneracy energy
3	Specific gravity	Gravitational force
4	Granulometry and granulomorphology	Minimum quantum of potential energy
5	Frictional couplings at the surface of the particles	Magnitude of the frictional forces

Condition (1) limits the range of applicability of the concepts stacking density, pressure in the loose material, and so on.

In statics, the particles of a loose material tend to occupy the lowest potential-energy levels as the coefficient of stacking density increases. This tendency is characterized by a steady loss of the ability to move relative to one another by the particles of the loose material.

As a consequence of the loss of ability to move, the loose material as a statistical aggregate becomes degenerate. The degeneracy is characterized by a transition to solid properties in the loose material, i.e., from discreteness to continuity.

In view of this we may regard the packing density as a characteristic of the mobility of the particles in the aggregate. The static characteristics of some of the loose material may therefore be considered as constituting a degeneracy of the statistical aggregate.

Changes in the packing density of the particles may be of a local nature, not extending over the whole of the volume.

This kind of density change is characterized by a local mobility of the particles, in which these lie outside any local change in packing density and remain immobile.

The considerable change in the packing density of the particles which occurs over the whole volume is associated with the global mobility of the particles, which is usually experienced when external forces act upon the whole of the loose material.

The foregoing properties of loose material have their analogs in a whole series of existing statistics. Under certain conditions we may therefore use the analytical apparatus developed for the statistics of loose materials in order to secure a theoretical generalization of the physical phenomena taking place in these. We must remember the capacity of these statistics to become degenerate in particular cases.

Allowing for the foregoing assumptions regarding the properties of loose material, we may now indicate its parameters (Table 1).

The foregoing parameters constitute either parameters of the solid phase (3, 4, 5), exerting the main influence on the state of the loose material, or else quantities which are in nature variable (1, 2), varying within specific limits. The latter determine the behavior of the loose material as a discrete statistical aggregate (degenerate in statics).

Like every statistical aggregate, loose material possesses specific properties. Whereas the properties of the phases constituting the loose material are described by their parameters, the properties of the loose material as a whole are of a far more complicated nature.

These properties are functions of both the parameters of the phases and the parameters of the statistical aggregate

$$\Phi_i = L_i(\langle x_i \rangle, \langle y_j \rangle). \tag{2}$$

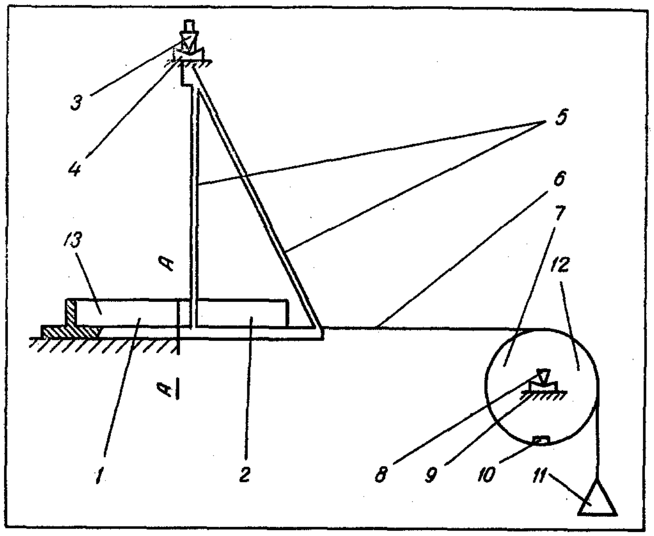

Fig. 2. General indication of the apparatus used for determining the specific adhesion.

The main properties of loose materials required for engineers' calculations are of a structural –mechanical, stress, and frictional nature.

The structural–mechanical properties of loose material appear in cases in which the degree of degeneracy of the statistical aggregate is high. In this case there may be a limiting transition to the continuous model of the loose medium. The structural–mechanical properties will be described by an equivalent elastic modulus and an equivalent Poisson coefficient.

For substantial specific pressures, the structural–mechanical properties of loose material are described by quantities characterizing the rheological model of Fig. 1.

The force properties characterize the transfer of energy in a closed volume containing the statistical aggregate of loose particles.

These properties include the practically established concepts of: a) the vertical pressure in the loose material; b) the horizontal pressure in the loose material; c) the vertical pressure on the bottom of the vessel; d) the horizontal pressure on the walls of the vessel; e) surfaces of equal pressure; f) coefficients of lateral thrust.

The character of energy transfer in loose material is largely determined by the frictional and adhesive forces at the contacts between its particles. The same forces determine the behavior of the loose material in the state in which the packing density changes from a certain maximum value K_{max} to a minimum K_{min}. These properties of loose material may arbitrarily be called frictional; they are determined by the angle of natural inclination or angle of internal friction at the surface of the loose material, the shear resistance of one layer of loose material relative to another, and the shear resistance of the loose material relative to the bounding surface.

The last two coefficients are complex quantities. For each specific loose material, under certain conditions, these may be resolved into components: 1) coefficients of friction (internal, external); 2) rupture resistance (within the layer or between the layer and the wall); 3) limiting shear resistance (layer on layer or layer on wall).

On the basis of the foregoing considerations, the effect of packing density on the physicomechanical properties of loose material acquires a first-order significance. The most correct approach to explaining the effect of packing density on the physicomechanical properties of the material is that of considering the loose medium as a discrete statistical aggregate.

For all statistical systems known at the present time, the Gibbs distribution (2), defining the probability of finding a subsystem of particles in a certain equilibrium state with energy E_n, holds true. This

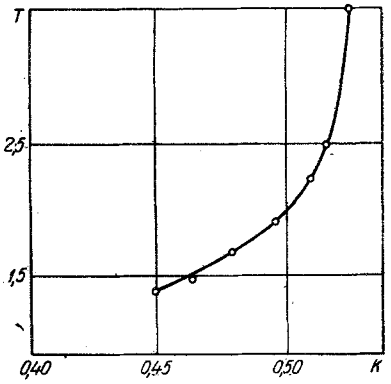

Fig. 3

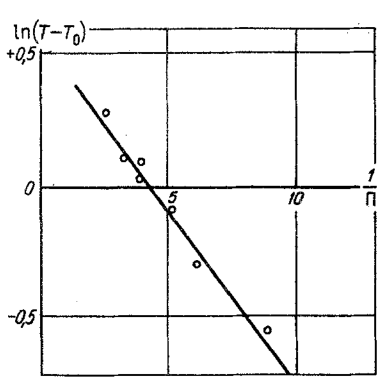

Fig. 4

Fig. 3. Dependence of the specific adhesion $T \cdot 10^2$ N/m^2 on the packing-density coefficient K for oxidized iron ore with a fractional composition of 0-1 mm.

Fig. 4. Dependence of $\ln(T - T_0)$ on $1/\Pi$.

relationship is determined by the well-known equation

$$\omega_n = C \exp\left(-\frac{E_n}{\Pi}\right). \tag{3}$$

Relating the energy of the particles in the statistical aggregate to their degree of mobility, we may assume that in the loose medium the packing density of its particles will affect their mobility in the same way as temperature. The packing density may in fact be considered as a statistical quantity appearing as a result of purely statistical laws and in general having no meaning if applied to nonmacroscopic systems.

Whereas a temperature may be considered as lying within the range (0, ∞) the packing density K of a loose medium exists in the range (K_{max}, K_{min}). The mobility of the particles ε varies over the range (0, ε_{max}). In order to complete the analogy between T and K, we must introduce a Π-criterion as proposed by Platonov [3]:

$$\Pi = \frac{K - K_{min}}{K_{max} - K}. \tag{4}$$

The Π-criterion varies over the range (0, ∞) as K varies over the range (K_{min}, K_{max}). In this case for a statistical aggregate of loose material Eq. (3) takes the form

$$\omega_n = C \exp\left(-\frac{b_k}{\Pi}\right). \tag{5}$$

Starting from the Gibbs distribution, we may assert that the probability of finding a particle in the state b_k, and hence the average number of particles n_k in this state, will be proportional to $\exp(-b_k/\Pi)$, i.e.,

$$n_k = a \exp\left(-\frac{b_k}{\Pi}\right). \tag{6}$$

Any quantity characterizing the state of the loose material $\varkappa$ will be proportional to the number of particles n_k, and we may thus write

$$\varkappa = \varkappa_0 + \nu \exp\left(-\frac{b}{\Pi}\right). \tag{7}$$

Here $\varkappa_0$ corresponds to $\Pi \to \infty$ and $K = K_{min}$.

Equation (7), describing the manner in which the properties of loose material depend on the packing density of their particles, was verified by determining the rupture resistance of connected loose materials as a function of their packing density. For this case Eq. (7) is written in the form

$$T = T_0 + (T_{max} - T_0) \exp\left(-\frac{b_k}{\Pi}\right). \tag{8}$$

The investigation was carried out in a special experimental apparatus illustrated schematically in Fig. 2.

The apparatus consists of the following parts: the left-hand half-pan 1, fixed firmly to the base, the right-hand half-pan 2, which is movable and is suspended on two prisms, which rest on the agate supports 4. The supports are fixed to the base. The pan 2 is rigidly fixed to the prisms 3 by means of the couplings 5. The suspension axis of the right-hand half-pan lies in the plane dividing the half-pans A – A. The distance between the line of the suspension and the half-pans is very significant. The half-pan 2 moves along the arc of a circle with a center lying on the line of the suspension; hence the greater the distance between the line of the suspension and the half-pans the smaller will the curvature be in the trajectory representing the motion of the half-pan 2. The line of the suspension should therefore be as high as possible. The filament 6 is joined to the right-hand half-pan 2 and passed over a pulley 7. The pulley 7 rests (via the prisms 8) on the agate supports 9, which are fixed to the base of the apparatus. The axis of the pulley coincides with the line of contact between the prisms 8 and the supports 9. In order to ensure that the pulley should be in a state of neutral equilibrium, a load 10 is fixed below the pulley.

To the second end of the filament a suspension 11 is fixed; this serves for loading the system. In order to avoid slipping of the filament 6 along 7, it is rigidly fixed to the pulley at point 12.

The experiment is carried out in the following order. Before the experiment the apparatus is calibrated by determining the force which has to be applied to the suspension 11 in order to remove the half-pan 2 from its place. Since the center of gravity of the half-pan 2 lies outside the plane of the suspension A – A, a moment develops, pressing it toward the half-pan 1 with a certain force, the value of which is determined in the calibration.

Then the inside 13 of the half-pans 1 and 2 is filled with finely dispersed loose test material having a specified packing density.

In order to avoid motion of the half-pan 2 while the loose material is being poured into it, it is fixed tightly to the half-pan 1 during this period. After filling with the material, half-pan 2 is carefully released from its coupling with half-pan 1. Then the suspension 11 is smoothly loaded, and from the force required to separate the half-pans from one another the rupture resistance is determined. This apparatus enables adhesions of the order of 10^3 N/m^3 to be measured.

The experiments were carried out with oxidized iron ore from the Krivorog basin with a granulometric composition of 0-6 mm and a 6.5% humidity. In order to create different packing densities, before each experiment the samples were compressed with a normal pressure of up to $1.5 \cdot 10^4$ N/m^2. Experimental data relating to the dependence of the rupture resistance on the packing density are presented in Fig. 3.

We see from Fig. 3 that, for low packing densities, the rupture resistance is insignificant; with increasing packing densities it increases rapidly. In calculating the criterion Π, we considered, in accordance with [3], that $K_{max} = 0.8$; K_{min} was found by direct experiment. The value of T corresponding to K_{min} was taken as T_0 in accordance with (6). Figure 4 shows the experimental relationship [$\ln(T - T_0)$; Π]. We see from the figure that the experimental points lie accurately on a straight line, the slope of which gives the value of b in (6). The point of intersection of the straight line with the vertical axis determines the ordinate equal to $\ln[T_{max} - T_0]$. The fact that the experimental points of the T(K) relationship in coordinates of [$\ln(T - T_0)$; 1/Π] lie on a straight line indicates the correctness of the statistical approach in studying the physicomechanical properties of loose materials.

Later it is proposed to verify Eq. (7) for a number of other properties of the loose material (internal and external friction, etc.).

NOTATION

Δl is the change in linear dimension;
ΔP is the change in specific pressure;
A is the constant;
L_i is the operator depending on the parameters of the loose aggregate x_i and the phase parameters y_i;
T is the temperature, the principal thermodynamic quantity determining the energy of the particles in the statistical aggregate;

b_k is the dimensionless parameter characterizing the potential barrier preventing the disruption of the equilibrium state of the loose particles;

a is the constant determining the normalization conditions;

N is the total number of loose particles;

T_0 is the rupture resistance at $K = K_{min}$;

T_{max} is the rupture resistance at $K = K_{max}$.

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